SmartShift: Expanded Load Shifting Incentive Mechanism for Risk-averse Consumers
Bochao Shen1, Balakrishnan Narayanaswamy2, Ravi Sundaram3
1College of Computer and Information Science, Northeastern University
2Department of Computer Science and Engineering, UCSD

Abstract
Peak demand for electricity continues to surge around the world. The supply-demand imbalance manifests itself in many forms, from rolling brownouts in California to power cuts in India. It is often suggested that exposing consumers to real-time pricing, will incentivize them to change their usage and mitigate the problem - akin to increasing tolls at peak commute times. We show that risk-averse consumers of electricity react to price fluctuations by scaling back on their total demand, leading to the unintended consequence of an overall decrease in production/consumption and reduced economic efficiency. We propose a new scheme that allows homes to move their demands from peak hours in exchange for greater electricity consumption in non-peak hours - akin to how airlines incentivize a passenger to move from an over-booked flight in exchange for, say, two tickets in the future. We present a formal framework for the incentive model that is applicable to smart grid forms of the electricity market. We show that our scheme not only enables increased consumption and consumer social welfare but also allows the distribution company to increase profits. This is achieved by allowing load to be shifted while insulating consumers from real-time price fluctuations. This win-win is important if these methods are to be embraced in practice.

Background – Pricing in electricity markets
• Electricity market participants:
  a) Distribution Companies – the suppliers of electricity
  b) Customers – consumers of electricity
• Prices seen by customers:
  a) Flat-rate pricing: de-facto standard for retail electricity
     i. High welfare for risk-averse consumers
     ii. cannot eliminate the peak load problem
  b) Real-time pricing:
     i. Peak reduction: by charging customers more when peak happens
     ii. But will discourage risk-averse consumers from buying due to the fluctuation nature.
• Prices seen by distribution company:
  a) Endogenous market price:
     i. Market price depends on the overall load
  b) Exogenous market price:
     i. Market price depends on one dominate (background) load.

Market model
• n end consumers:
  a) one distribution company (DC) pays $\omega_t$ in time slot t.
  b) Assumes demand and DC know the statistical property of the market price: the mean $\mu_{\omega}$ and the standard deviation $\sigma_{\omega}$.
• At the beginning of each billing cycle:
  a) DC will know the market price $\omega_t$.
  b) DC will decide the charging price $p_t$.
  1. Real-time pricing: $p_t = \omega_t$.
  2. Flat-rate pricing: $p_t = \omega_{\text{avg}}$.
• Risk-averse consumers:
  a) Valuation function: $v_i = w_i - \eta \sigma_i^2$.
  b) Utility function: $U_i = E_i - \frac{1}{2} \sigma_i^2$.
  c) Electricity consumption, social welfare and profit:
    i. Maximized Consumption: $\gamma_i^{\text{max}} = \frac{v_i}{\eta \sigma_i^2}$.
    1. Real-time pricing: $\gamma_i^{\text{max}} = \frac{v_i}{\eta \sigma_i^2}$.
    2. Flat-rate pricing: $\gamma_i^{\text{max}} = \frac{v_i}{\eta \sigma_i^2}$.

Optimal load shifting
• Endogenous case:
  a) Social welfare of consumers: the sum of net utilities $\sum_i (v_i - w_i - \eta \sigma_i^2)$
  b) Profit of DC: $\sum_i (v_i - w_i - \eta \sigma_i^2) - \sum_i p_t \gamma_i^{\text{max}}$
  c) $\eta < 0$: means that DC incurs loss with respect to its expected margins;
  d) $\eta > 0$: means that DC profits.

Exogenous market price: real time vs flat-rate
Theorem 1. With the same payment $p_t$, consumer $i$ will have no decrease in utility, if she shifts $\gamma_i^{\text{max}}$ from time slot t to time slot t with expansion, which results in an expanded load of $\omega_t = \omega_{\text{avg}} + \eta \sigma_i^2$ at time slot t. The necessary and sufficient condition, therefore is for the expansion ratio $m_{\omega} = \frac{\omega_i}{\omega_{\text{avg}}} < \frac{\gamma_i^{\text{max}}}{\eta}$ to satisfy the following:

$$m_{\omega} = \max \left( \frac{\gamma_i^{\text{max}}}{\eta} - 1 \right)$$

What is the benefit to the distribution company?
• The distribution company will agree to shift consumer i’s load from slot s to slot t only if it benefits from the shift.

Optimal load shifting
• Exogenous case:
  a) Exogenous market price:
  b) Market price depends on one dominate (background) load.

SmartShift: an alternative, expanded load shifting, incentive mechanism that addresses the challenges of peak demand and the supply-demand imbalance.
• What is the incentive for consumers to shift load?
  a) Each rational consumer $i$ will only agree to shift her consumption if her valuation does not decrease.
  b) Each end consumer $i$ will have no decrease in utility, if she shifts $\gamma_i^{\text{max}}$ from time slot s to time slot t with expansion, which results in an expanded load $\omega_t = \omega_{\text{avg}} + \eta \sigma_i^2$ at time slot t. The necessary and sufficient condition, therefore is for the expansion ratio $m_{\omega} = \frac{\omega_i}{\omega_{\text{avg}}} < \frac{\gamma_i^{\text{max}}}{\eta}$ to satisfy the following:

$$m_{\omega} = \max \left( \frac{\gamma_i^{\text{max}}}{\eta} - 1 \right)$$

Conclusion
We present SmartShift, a general incentive-based mechanism for reducing the load on the electricity grid. It grants users increased consumption in exchange for reducing their usage in peak periods. We show analytically that SmartShift under flat-rate pricing is a win-win for both consumers (increased social welfare) and producers (enhanced profits).

Exogenous market price: real time vs flat-rate
Theorem 4. Given exogenous market prices, the expected revenue $R_{\text{RT}}$ in real-time pricing is less than the expected revenue $R_{\text{FR}}$ in flat-rate pricing.

Simulation
We evaluate SmartShift using real-time electricity load data set from Smart* (Barker et al. 2012). We draw the risk aversion parameter $\lambda$ and expansion ratio ($\gamma_i^{\text{max}}$) from pareto distributions,

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Figure 1: Normalized revenue vs price volatility
• Figure 1 shows while real-time pricing induces risk-averse consumers to contract usage with increased volatility, SmartShift increases total revenue.

Figure 2: Loss probability vs consumer tolerance
• Figure 2 shows the less expansion the consumer adds, the less the probability of the distribution company incurring a negative profit ($\eta < 0$).

Figure 3: Probability density function of profit
• Figure 3 shows the probability mass of profit moves toward the positive direction right after SmartShift is applied.

References